Advanced Calculus **Summer Assignment**

Name Answer Key **Date**

Show all work! Exact answers only unless the problem asks for an approximation.

These are important topics from previous courses that you must be comfortable doing before you can be successful in Advanced Calculus.

If you find that you need some assistance, please feel free to Google or YouTube the concepts. Within the directions of each problem, you will see key words that you should be using in your research. I also recommend that you use the Desmos app for your phone or desmos.com on your computers to analyze the graph of functions if you do not have a graphing calculator.

1)Solve the linear equation.

$$5x-3(3x+1)-8x=45$$

$5x-9x-3-8x=45$

$-12x=48$

$x=-4$

2) Solve the linear equation.

$$4(x-8)-6x=3x-x-8$$

$4x-32-6x=2x-8$

$-2x-32=2x-8$

$-4x=24$

$x=-6$

3) Solve the equation algebraically.

$$24\left(7-\frac{x}{8}=\frac{x}{6}\right)$$

$168-3x=4x$

$168=7x$

$x=24$

4) Solve the equation algebraically.

$$\frac{2x+5}{4}=\frac{x-2}{3}$$

$6x+15=4x-8$

$2x=-23$

$x=-23/2=-11.5$

5) Solve the inequality, write the solution in interval notation & graph the solution set.

$$2(3-x)\geq 8$$

$6-2x\geq 8$

$-2x\geq 2$

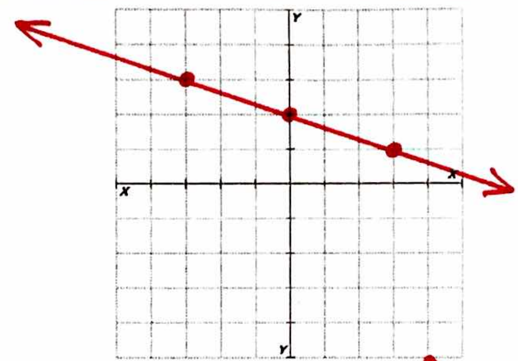
$x\leq -1$

-1

Interval Notation $(-\infty,-1]$

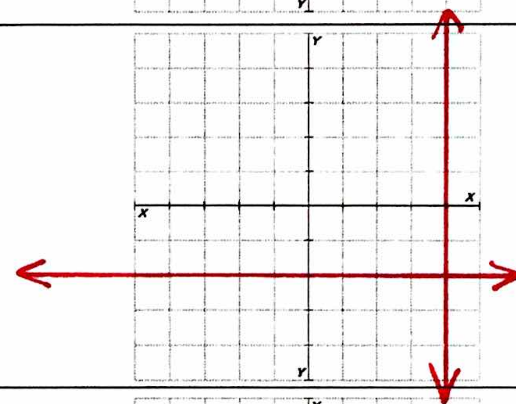
6) Graph the linear function.

$$y = -\frac{1}{3}x + 2$$



7) Graph the linear functions.

$x = 4$ and $y = -2$

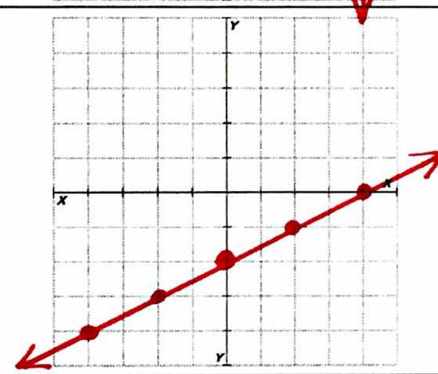


8) Graph the linear function.

$3x - 6y = 12$ $(4,0)$ $(0,-2)$

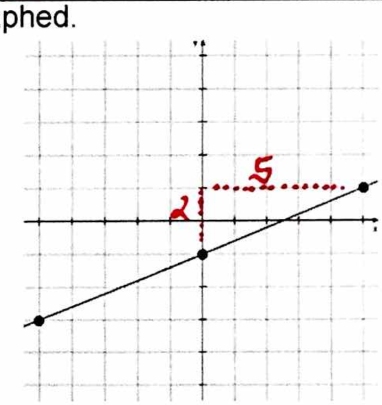
$-6y = -3x + 12$

$y = \frac{1}{2}x - 2$



9) Write the equation in slope-intercept form for the line that is graphed.

$y = \frac{2}{5}x - 1$



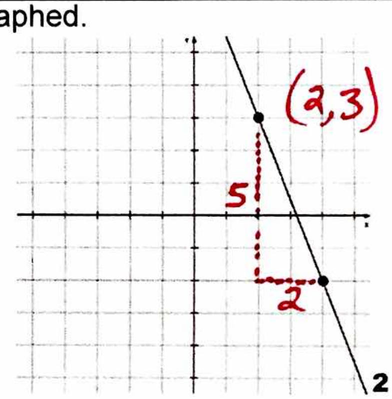
10) Write the equation in slope-intercept form for the line that is graphed.

$y = mx + b$

$3 = -\frac{5}{2} \cdot 2 + b$ $y = -\frac{5}{2}x + 8$

$3 = -5 + b$

$8 = b$



WILLEMS

11) Find the slope-intercept equation for a line with the given properties.

$Slope = \frac{3}{4}$ & containing the point $(-8,-16)$

$-16 = \frac{3}{4} \cdot -8 + b \rightarrow -16 = -6 + b \rightarrow -10 = b$

$y = \frac{3}{4}x - 10$

12) Find the slope-intercept equation for a line with the given properties.

Containing the points $(4,-4)$ & $(-6,21)$

$\frac{21 - -4}{-6 - 4} = \frac{25}{-10} = -5/2$ $\quad -4 = -\frac{5}{2} \cdot 4 + b$ $\quad b = 6$

$-4 = -10 + b$

$y = -\frac{5}{2}x + 6$

13) Find the slope-intercept equation for a line with the given properties.

Parallel to $y = -2x + 7$ & containing the point $(-7,6)$

$6 = -2 \cdot -7 + b$ $\quad b = -8$

$6 = 14 + b$

$y = -2x - 8$

14) Find the slope-intercept equation for a line with the given properties.

Perpendicular to $y = 4x - 13$ & containing the point $(-8,9)$

$9 = -\frac{1}{4} \cdot -8 + b \rightarrow 9 = 2 + b$

$7 = b$

$y = -\frac{1}{4}x + 7$

15) Simplify using properties of exponents.

$\left(-4xy^7\right)^2\left(-3x^5y^2\right) = 16x^2y^{14} \cdot -3x^5y^2 = -48x^7y^{16}$

16) Simplify using properties of exponents.

$\frac{-8x^{-3}y^{-7}z^5}{12x^{-1}y^{10}z^{-6}} = \frac{-8xz^5z^6}{12x^3y^{10}y^7} = \frac{-2z^{11}}{3x^2y^{17}}$

17) Rewrite the expression using rational exponent notation.

$\sqrt[6]{x^4} = x^{4/6} = x^{2/3}$

18) Rewrite the expression using radical notation.

$7^{\frac{1}{5}} = \sqrt[5]{7^1} = \sqrt[5]{7}$

19) Evaluate the expression without using a calculator.

$3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

20) Evaluate the expression without using a calculator.

$64^{\frac{2}{3}} = \sqrt[3]{64^2} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

21) Evaluate the expression without using a calculator.

$36^{-\frac{3}{2}} = \frac{1}{\left(\sqrt[2]{36}\right)^3} = \frac{1}{6^3} = \frac{1}{216}$

WILLEMS

22) Multiply the polynomials.

$$\left(x^2-3x-4\right)\left(5x^2+2x-1\right)$$

$$\begin{array}{r} 5x^4+2x^3-x^2 \\ -15x^3-6x^2+3x \\ -20x^2-8x+4 \\ \hline 5x^4-13x^3-27x^2-5x+4 \end{array}$$

23) Factor the polynomial.

$$2x^2-5x-12 = (2x+3)(x-4)$$

24) Factor the polynomial.

$$3x^2-37x+12 = (3x-1)(x-12)$$

25) Factor the polynomial.

$$4x^2+20x+25 = (2x+5)(2x+5) = (2x+5)^2$$

26) Factor the polynomial.

$$9x^2-100 = (3x+10)(3x-10)$$

27) Factor the polynomial.

$$2x^2-14x+24 = 2(x^2-7x+12) = 2(x-3)(x-4)$$

28) Factor the polynomial.

$$3x^3-12x = 3x(x^2-4) = 3x(x-2)(x+2)$$

29) Graph the quadratic function.

$$y=x^2-4x+3 = (x-1)(x-3)$$

Identify the following:

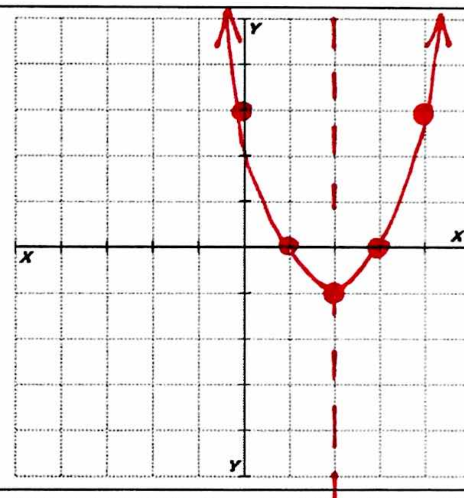
Direction of Opening UP

Y-intercept $(0,3)$

$x=\frac{-B}{2A}$ Axis of Symmetry $x=2$

Vertex $(2,-1)$

X-intercept(s) $(1,0)(3,0)$



30) Graph the quadratic function.

$$y=-3x^2-6x-1$$

Identify the following:

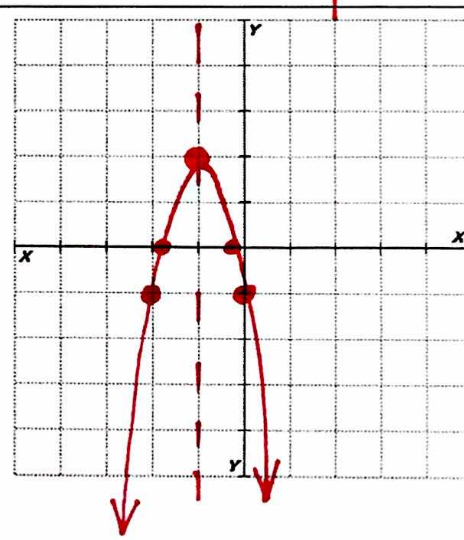
Direction of Opening DOWN

Y-intercept $(0,-1)$

$x=\frac{-B}{2A}$ Axis of Symmetry $x=-1$

Vertex $(-1,2)$

$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$ X-intercept(s) $(-1.82,0)(-0.18,0)$



WILLEMS

31) Solve the quadratic equation.

$$x^2 = -2x - 5$$

$x^2+2x+5=0$

NOT FACTORABLE

$x = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$

32) Solve the quadratic equation.

$$3x^2 - 20x - 32 = 0$$

$(3x+4)(x-8)=0$

$x = -\frac{4}{3}$ $x=8$

33) Solve the quadratic equation.

$$(x-5)^2 = 18$$

$x-5 = \pm\sqrt{18}$

$9 \cdot 2$

$x = 5 \pm 3\sqrt{2}$

34) Solve the square root equation (be sure to check for extraneous solutions).

$$\sqrt{7-x} - x = 5$$

$\sqrt{7-x} = x+5$

$7-x = x^2+10x+25$

$0 = x^2+11x+18$

$0 = (x+2)(x+9)$

$x=-2$ ✓ $x=-9$ EXTRANEOUS

35) Evaluate the function notation.

$$h(x) = 4x^2 - 3x + 11$$

$36+9+11$

Find $h(-3) =$ 56

ANSWER

$(-3, 56)$

36) Evaluate the function notation.

$$f(x) = \frac{x^3 - 337}{2x - 11}$$

$\frac{6}{3}$

Find $f(7) =$ 2

ANSWER

$(7, 2)$

37) Find the value of x in the function notation equation.

$$g(x) = -\frac{3}{5}x + 4$$

Find x *so that* $g(x) = 10$

$10 = -\frac{3}{5}x + 4$

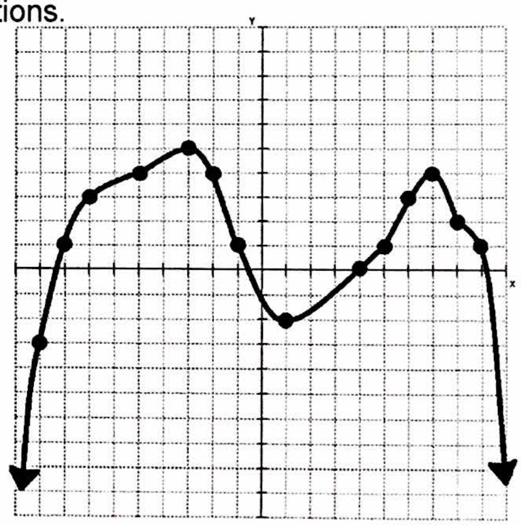
$6 = -3/5\, x$

$x = 6 \cdot -5/3 = -10$

ANSWER

$(-10, 10)$

38) Use the graph of $p(x)$ to answer the following questions.



A) *Find* $p(-9)$ $= -3$

B) *Find* $p(4)$ $= 0$

C) *Find* $p(8)$ $= 2$

D) *Find* x *so that* $p(x) = 5$

$x = -3$

E) *Find* x *so that* $p(x) = 4$

$x = -5, -2, 7$

F) *Find* x *so that* $p(x) = 1$

$x = -8, -1, 5, 9$

WILLEMS

39) Find the domain and range of the function in interval notation.

$$f(x)=\frac{3x^2-18}{x^2-9} = \frac{3x^2-18}{(x+3)(x-3)}$$

Domain $(-\infty,-3)\cup(-3,3)\cup(3,\infty$ Range $(-\infty,2]\cup(3,\infty)$

40) Find the domain and range of the function in interval notation.

$$g(x)=4\sqrt{2-x}$$

Domain $(-\infty,2]$ Range $[0,\infty)$

41) Let $f(x)=2x-5$, $g(x)=-x^2+3x$ and $h(x)=\sqrt{x-1}$. Perform the indicated operation.

A) $g(h(10))=$ 0 $\quad g(3)=0$

OUT IN

B) $f(g(-4))=$ -61 $\quad f(-28)=-61$

OUT IN

C) $f(g(x))=$ $-2x^2+6x-5$ $\quad f(-x^2+3x)=2(-x^2+3x)-5=-2x^2+6x-5$

OUT IN

D) $g(f(x))=$ $-4x^2+26x-40$ $\quad g(2x-5)=-(2x-5)^2+3(2x-5)=-(4x^2-20x+25)+6x-15=$
$-4x^2+20x-25+6x-15=-4x^2+26x-40$

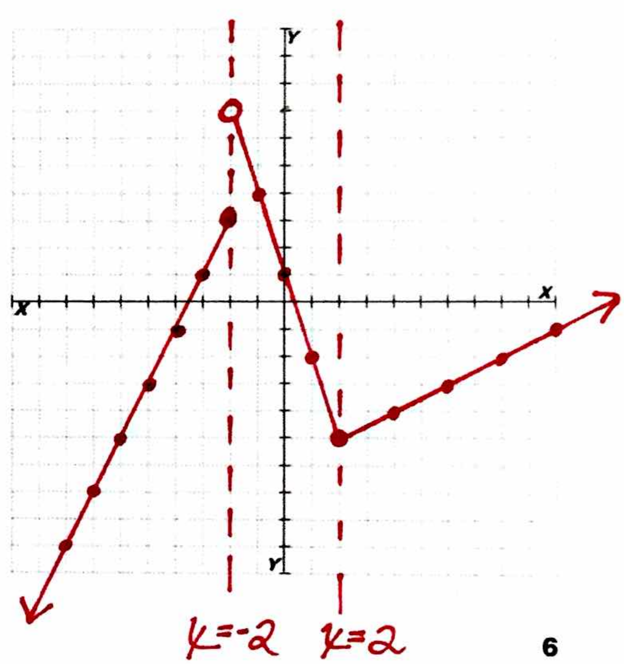
OUT IN

$y=2x-5$ E) $f^{-1}(x)=$ $\frac{x+5}{2}$ $\quad x=2y-5 \rightarrow x+5=2y \rightarrow y=\frac{x+5}{2}$

$y=\sqrt{x-1}$ F) $h^{-1}(x)=$ x^2+1 $\quad x=\sqrt{y-1} \rightarrow x^2=y-1 \rightarrow x^2+1=y$

42) Graph the following piecewise linear function.

$$g(x)=\begin{cases}2x+7, & x\le -2\\ -3x+1, & -2<x\le 2\\ \frac{1}{2}x-6, & x>2\end{cases}$$



A) Find $g(-4)=$ -1

B) Find $g(8)=$ -2

C) Find x so that $g(x)=-5$

$x=-6,2$

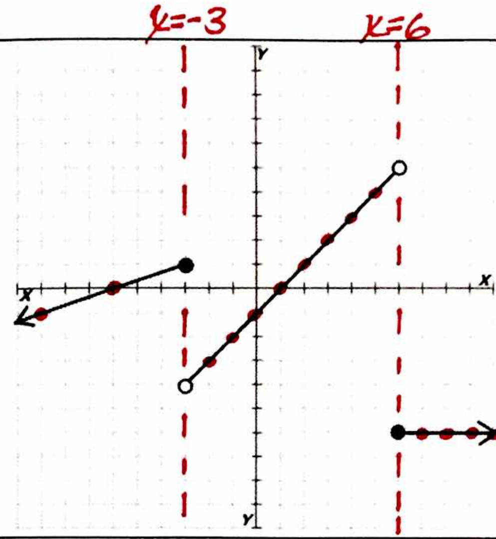
D) Find x so that $g(x)=1$

$x=-3,0$

WILLEMS

43) Write a piecewise function $h(x)$ for the graph.

$$h(x)=\begin{cases} \frac{1}{3}x+2, & x\le -3 \\ x-1, & -3<x<6 \\ -6, & x\ge 6 \end{cases}$$



For the polynomial functions (in factored form), list each real zero & its multiplicity, determine whether the graph crosses or touches the x-axis at each x-intercept, determine the end behavior model and then graph without using a calculator.

44) $y=-(x-2)(x-6)(x+3)$

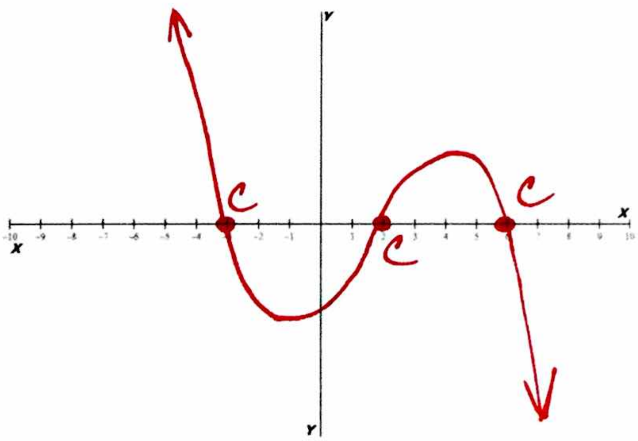
Zeros (include multiplicity & if the graph crosses or touches)

-3 MULT=1 CROSSES

2 MULT=1 CROSSES

6 MULT=1 CROSSES

End behavior model $y=-x^3$



45) $y=(x-3)^2(x-7)(x+2)^2$

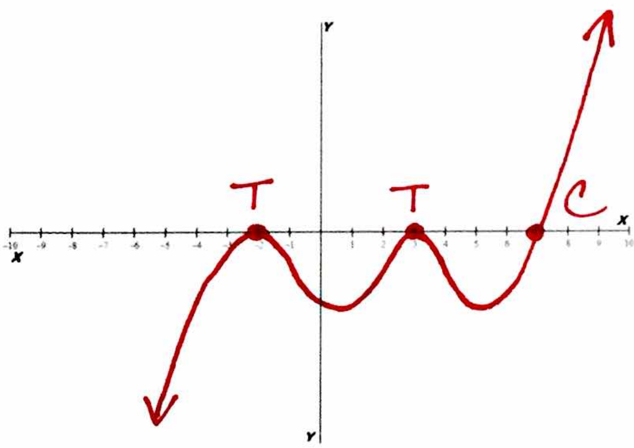
Zeros (include multiplicity & if the graph crosses or touches)

-2 MULT=2 TOUCHES

3 MULT=2 TOUCHES

7 MULT=1 CROSSES

End behavior model $y=x^5$



46) Find all local maxima & minima of the polynomial function. Then identify the interval(s) on which the function is increasing or decreasing in interval notation.

$$y=\frac{1}{4}x^4-\frac{1}{3}x^3-\frac{9}{2}x^2+9x+17$$

Maxima $(1, 21.417)$

Minima $(-3,-21.25)\ (3, 14.75)$

Increasing $[-3,1]\cup[3,\infty)$

Decreasing $(-\infty,-3]\cup[1,3]$

WILLEMS

47) Perform the indicated operation and simplify.

$$\frac{5}{x^2y}+\frac{-7x}{5y^2} = \frac{25y}{5x^2y^2}+\frac{-7x^3}{5x^2y^2} = \frac{25y-7x^3}{5x^2y^2}$$

48) Perform the indicated operation and simplify.

$$\frac{x}{\underset{(x-2)(x+2)}{x^2-4}}+\frac{2}{\underset{3(x+2)}{3x+6}} = \frac{3x}{3(x+2)(x-2)}+\frac{2x-4}{3(x+2)(x-2)} = \frac{5x-4}{3(x+2)(x-2)}$$

49) Perform the indicated operation and simplify.

$$\frac{y}{y+3}+\frac{-6y}{\underset{(y+3)(y-3)}{y^2-9}} = \frac{y^2-3y}{(y+3)(y-3)}+\frac{-6y}{(y+3)(y-3)} = \frac{y^2-9y}{(y+3)(y-3)} = \frac{y(y-9)}{(y+3)(y-3)}$$

50) Solve the equation.

$$12y\left(\frac{2}{3y}+\frac{5}{6y}=\frac{3}{4}\right) \rightarrow 8+10=9y$$

$$18=9y \qquad y=2$$

51) Solve the equation.

$$(x-2)(x+2)\left(\frac{x-4}{x-2}=\frac{x-2}{x+2}+\frac{1}{x-2}\right) \rightarrow x^2-4x+2x-8=x^2-2x-2x+4+x+2$$

$$x^2-2x-8=x^2-3x+6$$

$$x=14$$

52) Solve the equation.

$$(x+2)(x+3)\left(\frac{3}{\underset{(x+2)(x+3)}{x^2+5x+6}}+\frac{x-1}{x+2}=\frac{7}{x+3}\right) \rightarrow 3+x^2-x+3x-3=7x+14$$

$$x^2-5x-14=0 \qquad x=7 \qquad x=-2$$

$$(x-7)(x+2)=0 \qquad \text{EXTRANEOUS}$$

Identify the hole(s), vertical asymptote(s), horizontal asymptote, oblique asymptote, x-intercept(s) & y-intercept of each rational function. Then graph each function.

53) $g(x)=\dfrac{-x^2+9x-8}{x^2+x-2} = \dfrac{-(x-8)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}$

Hole(s) $\frac{7}{3}$ $(1, 7/3)$

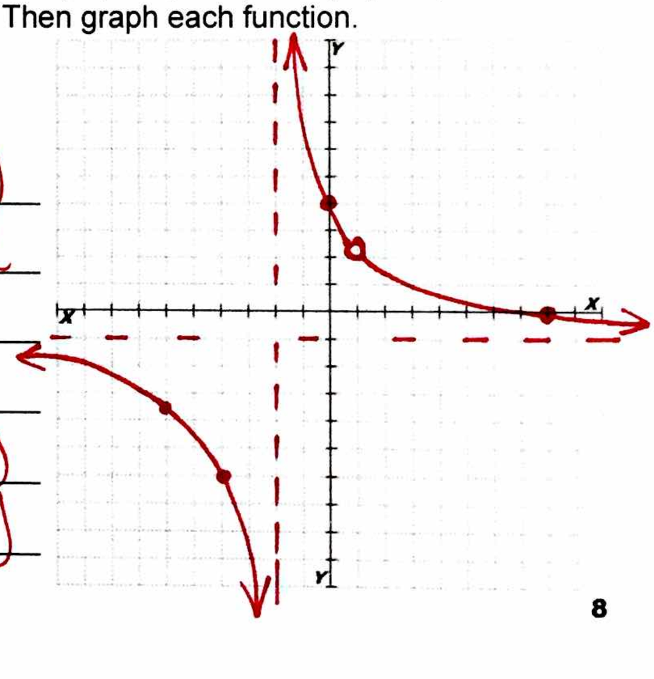
Vertical asymptote(s) $x=-2$

Horizontal asymptote $y=-1$

Oblique asymptote NONE

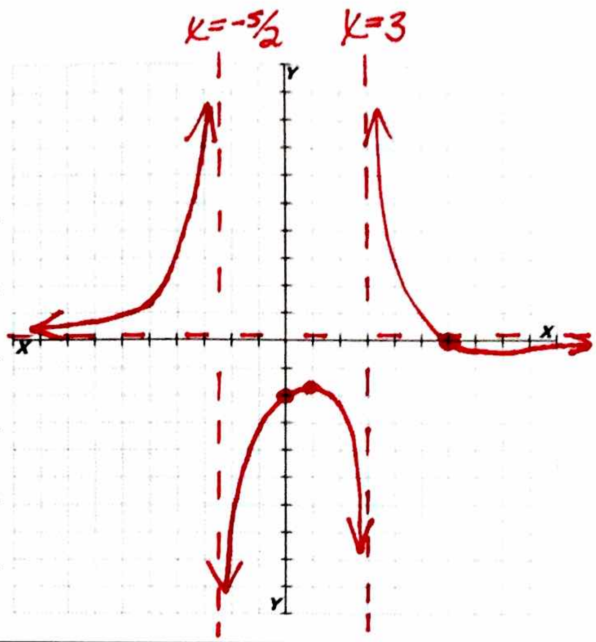
X-intercept(s) $(8, 0)$

Y-intercept $(0, 4)$



WILLEMS

54) $h(x) = \dfrac{-5x+30}{2x^2-x-15} = \dfrac{-5(x-6)}{(2x+5)(x-3)}$



Hole(s) NONE

Vertical asymptote(s) $x=-5/2,\ x=3$

Horizontal asymptote $y=0$

Oblique asymptote NONE

X-intercept(s) $(6,0)$

Y-intercept $(0,-2)$

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55) Solve the inequality and graph the solution set. Write the solution in interval notation.

$-(x-5)(x+2)(x-1)<0$

NEG

Interval Notation $(-2,1)\cup(5,\infty)$

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56) Solve the inequality and graph the solution set. Write the solution in interval notation.

$x^4+16x^2>10x^3$

$x^4-10x^3+16x^2>0$

$x^2(x^2-10x+16)>0 \longrightarrow x^2(x-2)(x-8)>0$

POS

Interval Notation $(-\infty,0)\cup(0,2)\cup(8,\infty)$

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57) Solve the inequality and graph the solution set. Write the solution in interval notation.

$\dfrac{x(x+5)}{x-7}\geq 0$

POS

Interval Notation $[-5,0]\cup(7,\infty)$

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58) Solve the inequality and graph the solution set. Write the solution in interval notation.

$\dfrac{(x-8)^2(x+3)}{(x-4)}\leq 0$

NEG

Interval Notation $[-3,4)$

Evaluate the logarithms without using a calculator.

59) $\log_{81}\left(\dfrac{1}{27}\right) = -3/4$ $\quad \frac{1}{27}=81^{?} \rightarrow 3^{-3}=(3^4)^{?} \rightarrow -3=4\cdot ? \rightarrow ?=-3/4$

60) $\ln_e\left(\sqrt{e^5}\right) = 5/2$ $\quad e^{5/2}=e^{?}$

WILLEMS

61) Graph the logarithmic function and determine its domain, range and asymptote(s). Use interval notation for the domain and range.

$f(x) = \log_2(x)$

$y = \log_2 x$

$\downarrow$

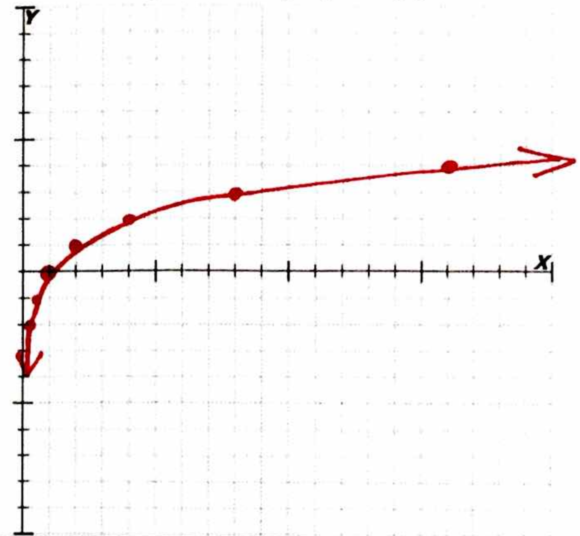
$x = 2^y$

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2

Domain $(0, \infty)$

Range $(-\infty, \infty)$

Asymptote(s) X=0 OR Y-AXIS



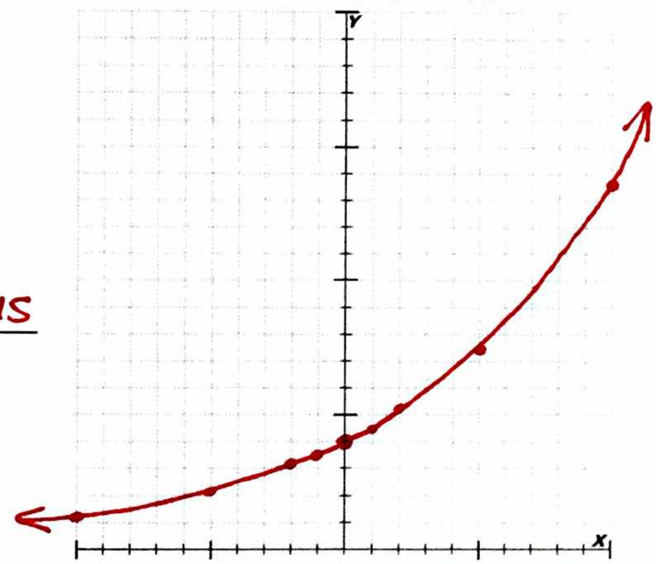
62) Graph the exponential function and determine its domain, range and asymptote(s). Use interval notation for the domain and range.

$g(x) = 4e^{0.123x}$

Domain $(-\infty, \infty)$

Range $(0, \infty)$

Asymptote(s) y=0 OR X-AXIS



Graph the implicitly defined equations.

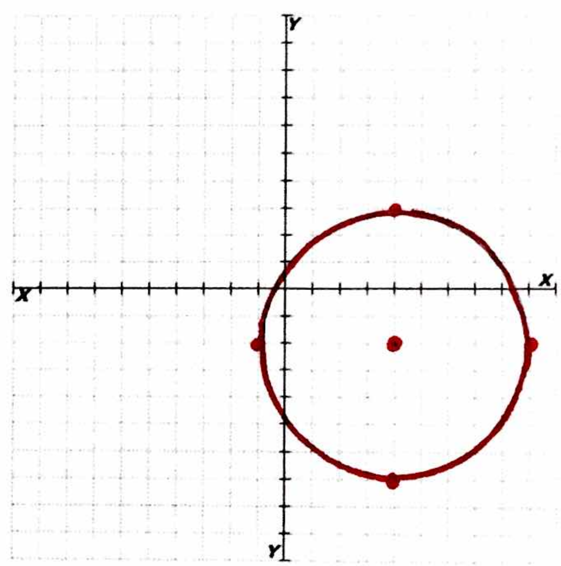
63) $(x-4)^2 + (y+2)^2 = 25$

CIRCLE

CENTER

(4, -2)

RADIUS = 5



WILLEMS

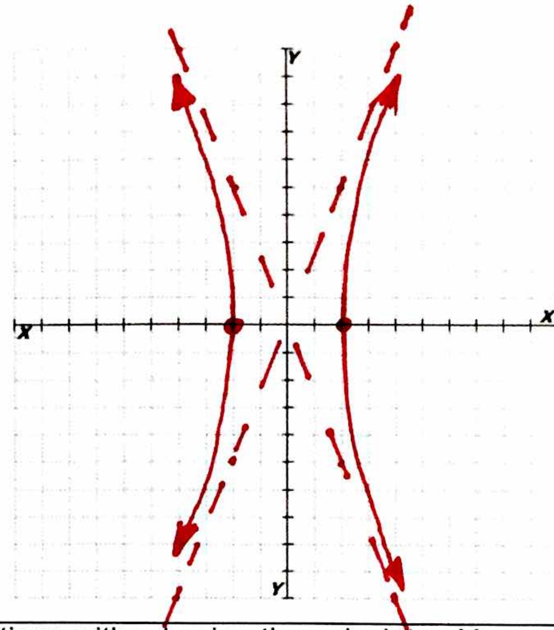
64) $\frac{x^2}{4} - \frac{y^2}{25} = 1$

$a=2 \quad b=5$

HYPERBOLA

ASYMPTOTES

$y = \pm\frac{5}{2}x$



65) Evaluate the exact value of the trigonometric functions without using the calculator. You are expected to have the unit circle memorized.

A) $\cos\left(\frac{3\pi}{2}\right) = $ 0

B) $\sec\left(-\frac{3\pi}{4}\right) = $ $-\sqrt{2}$

C) $\sin\left(\frac{13\pi}{6}\right) = $ $\frac{1}{2}$

D) $\tan\left(-\frac{15\pi}{4}\right) = $ 1

E) $\csc\left(-\frac{10\pi}{3}\right) = $ $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

F) $\cot\left(\frac{35\pi}{6}\right) = $ $-\sqrt{3}$

66) Evaluate the exact value in radians of the inverse trigonometric functions without using a calculator. You are expected to have the unit circle memorized.

A) $\sin^{-1}(-1) = $ $-\frac{\pi}{2}$

B) $\cos^{-1}(0) = $ $\frac{\pi}{2}$

C) $\tan^{-1}(1) = $ $\frac{\pi}{4}$

D) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = $ $-\frac{\pi}{4}$

E) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = $ $\frac{5\pi}{6}$

F) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = $ $-\frac{\pi}{6}$

WILLEMS